RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.SC. FIRST SEMESTER EXAMINATION, DECEMBER 2012

FIRST YEAR

PHYSICS (Honours)

Date : 14/12/2012

Time : 11 am – 2 am

Paper : I

Full Marks : 75

Group – A

(*Answer **any five** questions*)

1. a) Find the interval of convergence for the series $x-\frac{x^2}{2}+\frac{x^3}{3}\cdots+\frac{(-1)^{n+1}}{n}x^n+\cdots$. [4]

   b) Test for the convergence of the series $\sum_{n=5}^{\infty}\frac{1}{2^n-n^2}$. [3]

   c) Let $f(x)$ be defined as, $f(x)=1$ for $x>0$
   $=0$ for $x\le 0$

   Hence prove that $\frac{df}{dx}=\delta(x)$. [3]

2. a) Prove by using Stokes theorem that $\text{Curl}(\vec{\nabla}\phi)=0$, where ϕ is any scalar. (no statement of the theorem is needed). [2]

   b) Expand $\frac{1}{1-x}$ about x = ½. [4]

   c) Five percent of the tools produced in a factory turn out to be defective. Find the probability that in a sample of 10 tools chosen at random, exactly two will be defective, using (i) Binomial distribution, (ii) Poisson distribution. [2+2]

3. a) Given $\vec{V}=x^2\hat{i}+y^2\hat{j}+z^2\hat{k}$, evaluate $\int_S \vec{V}\cdot\hat{n}\,d\sigma$ where S is the surfaces of the cube with vertices at (0, 0, 0), (0, 1, 0), (0, 0, 1), (1, 0, 0).
   Evaluate $\int_v \vec{\nabla}\cdot\vec{V}\,dv$, where volume integral is over the volume of the above cube. [6]

   b) Given $u=x^2+y^2,\ y=x^z$, find $\frac{\partial u}{\partial x}$ if (i) x, y are independent and (ii) x, z are independent. [2]

   c) Prove $\vec{\nabla}\cdot(\vec{A}\times\vec{B})=\vec{B}\cdot(\vec{\nabla}\times\vec{A})-\vec{A}\cdot(\vec{\nabla}\times\vec{B})$. [2]

4. a) Consider the matrix $A=\begin{pmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{pmatrix}$

   Find the eigenvalues and construct the diagonalising matrix. [7]

   b) Find out $\hat{e}_r,\hat{e}_\theta,\hat{e}_\phi$ in terms of the unit vectors $\hat{i},\hat{j},\hat{k}$ of a Cartesian coordinate system. Hence prove that the spherical polar coordinate system is an orthogonal system. [3]

5. a) Find the Fourier series for the periodic function defined by $f(x)=e^x,\ -\pi\le x<\pi$.

   Using the series show that $\sum_{1}^{\infty}\frac{(-1)^n}{n^2+1}=\frac{1}{2}\left[\frac{\pi}{\sinh\pi}-1\right]$. [6]

   b) Explain the difference between a polar and an axial vector with reference to their behaviour under transformation of coordinates. [4]

6. a) Solve by power series expansion $y'' + xy = 0$ (First 4 terms are sufficient). [4]

b) Find the steady state temperature (T) distribution in a metal plate 10 cm square, if one side is held at 100ºC and the other sides at 0ºC by solving Laplaces equation $\nabla^2 T = 0$. [6]

7. a) The differential equation $\dfrac{d^2y}{dx^2} - 2x\dfrac{dy}{dx} + 2xy = 0$ has the solution of the form $y = \sum_n a_n x^{k+n}$. Find k and the recursion relation between the coefficients. [5]

b) Assuming $e^{2tx - t^2} = \sum_{n=0}^{\infty} \dfrac{t^n}{n!} H_n(x)$, show that $\int_{-\infty}^{\infty} e^{-x^2} H_m(x) H_n(x)\, dx = 2^x n! \sqrt{\pi}\, \delta_{mn}$. [5]

8. a) If $AB = BA$, prove that A and B are square matrices with the same dimension. [3]

b) If $Ax = \lambda x$, and A is an orthogonal matrix, prove that A^{-1} has the same eigenvector X with the eigenvalue λ^{-1}. [3]

c) Given $x^2 u - y^2 v = 1$ and $x + y = uv$, find $\left.\dfrac{\partial x}{\partial u}\right|_v$ and $\left.\dfrac{\partial x}{\partial u}\right|_y$. [4]

Group – B

(*Answer* **any two** *questions*)

9. a) Construct a suitable matrix to represent the effect of refraction by a curved refracting surface. [2]

b) Deducing the system matrix for thick lens, derive an expression for the focal length of a thin convex lens under paraxial approximation. [4+2]

c) Show that the planar surface of a plano-convex lens does not contribute to the system matrix. [2]

10. a) Define principal points and nodal points of an optical system. Show that the distance between first principal point and nodal point is equal to distance between second principal point and nodal point. [2+4]

b) What do you mean by aplanatic points and aplanatic surface? What are the advantages of aplanatic surfaces? [1+1+2]

11. a) Define linear and longitudinal magnification. Deduce a relation between them. [2+2]

b) Two thin convex lenses having focal lengths 5 cm and 2 cm are placed coaxially and separated by a distance 3 cm. Find the equivalent focal length and positions of principal and nodal points. [4]

c) Show that in a telescopic system the distance between the two lenses is equal to the sum of their focal lengths. [2]

12. Give the construction and explain the working of a Ramsden eye piece with the help of a ray diagram. How are chromatic and spherical aberrations minimised in this eye piece? Make the necessary calculations. [2+2+3+3]

Answer **any one** question: 1x5

13. State and explain Fermat's principle. Using this principle, establish the laws of reflection on a concave surface. [2+3]

14. a) Show that when the media on the two sides of a lens are same the nodal points coincide with the principal points. [3]

b) What do you mean by "entrance and exit pupils" of an optical instrument? [2]